Exercise 1

Show that the following argument is valid using the tableau method:

$$\forall x(\exists y\, Rxy \rightarrow Rxfx)$$
$$\forall x\, \exists y\, Rxy$$
$$\exists x\, Rffxx$$

$$\exists x\, \exists y\, \exists z\ (Rxy \wedge Ryz \wedge Rzx)$$

Exercise 2

Determine whether the following set of sentences is consistent and, if yes, describe a model of the set:

$$A: \quad \exists x\, \forall y\ (Px \wedge (Ry \rightarrow Qxy))$$
$$\forall x\, \forall y\ (\neg Px \vee \neg Sy \vee \neg Qxy)$$
$$\exists y\ (Ry \wedge Sy)$$

Exercise 3

Consider the following set of sentences

$$\mathbb{A}: \quad \forall x \forall y \forall z \; ffxyz = fxfyz$$
$$\exists x \forall y \exists z \left(fyx = y \wedge fyz = x \right)$$

For each of the following three sentences, decide whether it is a consequence of $\mathbb{A}$ and, if not, give a model in which $\mathbb{A}$ holds, but the sentence fails.

$$F_1: \quad \forall x \forall y \forall z \left(fxz = fyz \longrightarrow x = y \right)$$

$$F_2: \quad \forall x \forall y \left(fxy = fyx \right)$$

$$F_3: \quad \forall y \forall y \forall z \left(fzx = fzy \longrightarrow x = y \right)$$